I Semester B.A./B.Sc. Examination, November/December 2018
(CBCS) (F + R)
(2014 – 15 & Onwards)
MATHEMATICS (Paper – I)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

(5×2=10)

1. Answer **any five** questions.

a) Define equivalent matrices.

b) Find the eigen values of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

c) Find the n^{th} derivative of $\cos^2 x$.

d) If $z = x^3 - 4x^2y + 5y^2$, find $\frac{\partial^2 z}{\partial x \, \partial y}$.

e) Evaluate $\int_0^{\pi/2} \sin^6 x dx$.

f) Evaluate $\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx$.

g) Find the angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane $x + y + z + 5 = 0$.

h) If the two sphere $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cuts orthogonally, find k.

PART – B

(1×15=15)

Answer **one full** question.

2. a) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 4 & 7 & 10 \end{bmatrix}$ by row reduced Echelon form.

b) Find the non-trivial solution of the system of equations $2x - y + 3z = 0$, $3x + 2y + z = 0$, $x - 4y + 5z = 0$.

c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$.

OR

3. a) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing into its normal form.

b) Show that the following system of equations are consistent and solve them $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$.

c) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

PART – C

Answer **two full** questions. **(2×15=30)**

4. a) Find the n^{th} derivatives of $\frac{2x-1}{(x+1)(x-2)}$.

b) Find the n^{th} derivative of (i) $\log(5x-1)$ (ii) $\cos 5x \cdot \cos 3x$.

c) If $y = (\sin^{-1}x)^2$, show that $(1 - x^2)y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0$.

OR

5. a) If $Z = \sin(ax + y) + \cos(ax - y)$. Prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

b) State and prove Euler's theorem for homogeneous functions.

c) Find $\frac{du}{dt}$, where $u = e^x \sin y$, $x = \log t$, $y = t^2$.

6. a) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$.

c) Obtain the reduction formula for $\int \sin^n x\, dx$, where n is a positive integer.

OR

7. a) Obtain the reduction formula for $\int \sec^n x\,dx$, where n is positive integer.

b) Evaluate $\int_0^{\pi} x\cos^6 x\,dx$.

c) Evaluate by using Leibnitz's rule of differentiation under the integral sign for $\int_0^{\pi/2} \frac{dx}{\alpha(1+\cos x)}$, where α is a parameter.

PART – D

Answer **one full** question. **(1×15=15)**

8. a) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 2z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z + 6 = 0$.

b) Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-3}{1}$ and $\frac{x-2}{2} = \frac{y-4}{1} = \frac{z-6}{3}$ are co-planar. Find the equation of the plane containing them.

c) Obtain the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and which has its centre on the plane $3x - y + z = 2$.

OR

9. a) Find the shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

b) Find the equation of the right circular cone whose vertex is at (2, –3, 5), axis makes equal angles with the co-ordinate axes and the semivertical angle is measured to be 30°.

c) Find the equation of the right circular cylinder for which radius 4, whose axis is the line $\frac{x-1}{2} = \frac{y-3}{-3} = \frac{z-3}{6}$.

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